Suffix Arrays

Algorithms for Sequence Analysis

Sven Rahmann

Summer 2021

Plan

Previous Lecture

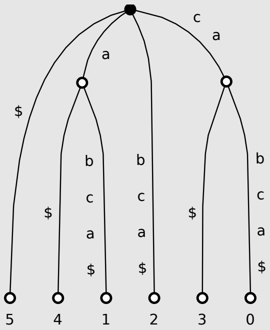
- Suffix trees
  - Basic applications
  - Linear time construction

Today

- Suffix arrays
- LCP arrays
- Applications
  - Pattern matching, longest repeated substring, shortest unique substring, longest common substring, maximal unique matches (MUMs)
- Longest common prefix (LCP) array construction

Suffix trees and suffix arrays

Suffix tree for the string $T = cabca\$$.



A suffix array of a string s\$ with $|s\$| = n$ is defined as the permutation *pos* of $\{0, .., n-1\}$ that represents the lexicographic ordering of all suffixes of s\$. Here $pos = [5, 4, 1, 2, 3, 0]$.

Motivation for Suffix Arrays

- High memory requirements for suffix tree ($O(n) \approx 20n$ bytes)
- With alphabetically sorted outgoing edges:
  Sequence of leaf numbers
  $=$ starting positions of lexicographically sorted suffixes
- Array: $O(4n)$ bytes (for 32-bit integers, $n < 2^{32}$)

Motivation for Suffix Arrays

- High memory requirements for suffix tree ($O(n) \approx 20n$ bytes)
- With alphabetically sorted outgoing edges:
  Sequence of leaf numbers
  $=$ starting positions of lexicographically sorted suffixes
- Array: $O(4n)$ bytes (for 32-bit integers, $n < 2^{32}$)
- Represents only the leaf level of the suffix tree
- Representation of tree structure with additional arrays
- Some questions can be solved directly with cache-efficient algorithms

Example of a Suffix Array

Notation: p for text positions, r for lexicographic ranks.
In a auffix array $\mathrm{pos}[r]$ is the text position where the r-th smallest suffix starts.

$p =$	0	1	2	3	4	5	6	7	8	9	10	11	12	13
$T =$	m	i	i	s	s	i	s	s	i	p	p	i	i	$

$r =$	0	1	2	3	4	5	6	7	8	9	10	11	12	13
$\mathrm{pos} =$	13	12	11	1	8	5	2	0	10	9	7	4	6	3

Example of a Suffix Array

Notation: p for text positions, r for lexicographic ranks.
In a auffix array $\mathrm{pos}[r]$ is the text position where the r-th smallest suffix starts.

$p =$	0	1	2	3	4	5	6	7	8	9	10	11	12	13
$T =$	m	i	i	s	s	i	s	s	i	p	p	i	i	\$

$r =$	0	1	2	3	4	5	6	7	8	9	10	11	12	13
$\mathrm{pos} =$	13	12	11	1	8	5	2	0	10	9	7	4	6	3
	\$	i						m	p		s			

We may partition the suffixes into “buckets” according to their first letter.

Construction of Suffix Arrays

Three possibilities

1. from the suffix tree by scanning the leaves, in $O(n)$ time
   Disadvantage: high memory consumption for intermediate tree

Construction of Suffix Arrays

Three possibilities

1. from the suffix tree by scanning the leaves, in $O(n)$ time
   Disadvantage: high memory consumption for intermediate tree
2. directly by some standard sorting algorithm

```python
def build_suffixarray_naive(T):
    suffixes = lambda p: T[p:]
    return sorted(range(len(T)), key=suffixes)
```

   Disadvantage: Running time

Construction of Suffix Arrays

Three possibilities

1. from the suffix tree by scanning the leaves, in $O(n)$ time
   Disadvantage: high memory consumption for intermediate tree
2. directly by some standard sorting algorithm

```python
def build_suffixarray_naive(T):
    suffixes = lambda p: T[p:]
    return sorted(range(len(T)), key=suffixes)
```

Disadvantage: Running time $O(n^2 \log n)$

Construction of Suffix Arrays

Three possibilities

1. from the suffix tree by scanning the leaves, in $O(n)$ time
   Disadvantage: high memory consumption for intermediate tree
2. directly by some standard sorting algorithm

```
def build_suffixarray_naive(T):
    suffixes = lambda p: T[p:]
    return sorted(range(len(T)), key=suffixes)
```

   Disadvantage: Running time $O(n^2 \log n)$
3. directly by an efficient linear-time algorithm (later)
   Disadvantage: complicated algorithm

Search with suffix arrays

Definitions

- Pattern $P \in \Sigma^m$ and text $T \in \Sigma^n$
- Define

$$L := \min \left[\{ r | P \leq T[\mathrm{pos}[r] \ldots] \} \cup \{n\} \right],$$
$$R := \max \left[\{ r | P \geq T[\mathrm{pos}[r] \ldots \mathrm{pos}[r] + |P|] \} \cup \{-1\} \right].$$

- All suffixes in the interval $[L, R]$ start with P
- P occurs in T if (and only if) $R \geq L$
- Searching in suffix array $\Longleftrightarrow$ determining $[L, R]$
- Use two **binary searches** to determine $[L, R]$.

Example: Binary search in Suffix Arrays

Search for "is", then for "sp".

$p =$	0	1	2	3	4	5	6	7	8	9	10	11	12	13
$T =$	m	i	i	s	s	i	s	s	i	p	p	i	i	\$

$r =$	0	1	2	3	4	5	6	7	8	9	10	11	12	13
$\mathrm{pos} =$	13	12	11	1	8	5	2	0	10	9	7	4	6	3

Running Time for Searching

1. **Decision problem**
   As we have seen, the running time is $\mathcal{O}(m \log n)$.
2. **How often does P occur in T?**
   Same as above, because the number of occurrences is $k = R - L + 1$.
3. **Where does P occur in T?**
   Once the interval $[L, R]$ is known, the start positions can be found by scanning through the interval in additional $\mathcal{O}(k)$ time.

Running Time for Searching

1. **Decision problem**
   As we have seen, the running time is $\mathcal{O}(m \log n)$.
2. **How often does P occur in T?**
   Same as above, because the number of occurrences is $k = R - L + 1$.
3. **Where does P occur in T?**
   Once the interval $[L, R]$ is known, the start positions can be found by scanning through the interval in additional $\mathcal{O}(k)$ time.

Note: With a different approach, the factor $\log n$ can be saved!

Motivation: Enhanced Suffix Arrays

Can we use suffix arrays just like suffix trees?

Motivation: Enhanced Suffix Arrays

Can we use suffix arrays just like suffix trees?
Not like defined so far.... We need more structure!

Motivation: Enhanced Suffix Arrays

Can we use suffix arrays just like suffix trees?
Not like defined so far. . . . We need more structure!

- Enhancing suffix arrays with **Longest Common Prefix (LCP)** arrays to represent the tree structure above the leaf level
- **Applications** of enhanced suffix arrays
  - Longest repeated substring
  - Shortest unique substring
  - Longest common substring
  - Maximal unique matches (MUMs)

Longest Common Prefix (LCP) arrays

LCP Array by Example

$p =$	0	1	2	3	4	5	6	7	8	9	10	11	12	13
$T =$	m	i	i	s	s	i	s	s	i	p	p	i	i	\$

$r =$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	
`pos` =	13	12	11	1	8	5	2	0	10	9	7	4	6	3	
`lcp` =	-1	0						0	0		0				-1
	\$	i						m	p		s				

`lcp` represents longest common prefixes
of lexicographically adjacent suffixes (looking left).

LCP Array

Definition: longest common prefix array

Let $T \in \Sigma^n$ be a text and let pos be the corresponding suffix array. We define `lcp` to be an array of length $(n+1)$ such that

$$\texttt{lcp}[r] = \begin{cases} -1 & \text{if } r = 0 \text{ or } r = n, \\ lcp\big(T[\text{pos}[r-1]\ldots], T[\text{pos}[r]\ldots]\big) & \text{otherwise,} \end{cases}$$

where

$$lcp(s,t) := \max\big\{ i \in \mathbb{N}_0 \mid s[\ldots i-1] = t[\ldots i-1] \big\}.$$

LCP Array

Definition: longest common prefix array

Let $T \in \Sigma^n$ be a text and let pos be the corresponding suffix array.
We define `lcp` to be an array of length $(n+1)$ such that

$$\texttt{lcp}[r] = \begin{cases} -1 & \text{if } r = 0 \text{ or } r = n, \\ lcp\big(T[\mathrm{pos}[r-1]\ldots], T[\mathrm{pos}[r]\ldots]\big) & \text{otherwise,} \end{cases}$$

where

$$lcp(s,t) := \max\left\{ i \in \mathbb{N}_0 \mid s[\ldots i-1] = t[\ldots i-1] \right\}.$$

Terminology

A suffix array plus (an) auxiliary array(s) like `lcp` is called **enhanced suffix array**.

Naive Construction of LCP Array

```python
def lcp_naive(pos,T):
    lcp = [-1]  # first -1 (at index 0)
    for r in range(1, len(T)):
        # compare suffix starting at pos[r-1]
        # to suffix starting at pos[r]
        x = 0
        while T[pos[r-1]+x] == T[pos[r]+x]:
            x += 1  # cannot run off the string (sentinel!)
        lcp.append(x)
    lcp.append(-1) # trailing -1 (at index n)
    return lcp
```

Naive Construction of LCP Array

```
def lcp_naive(pos,T):
    lcp = [-1]  # first -1 (at index 0)
    for r in range(1, len(T)):
        # compare suffix starting at pos[r-1]
        # to suffix starting at pos[r]
        x = 0
        while T[pos[r-1]+x] == T[pos[r]+x]:
            x += 1  # cannot run off the string (sentinel!)
        lcp.append(x)
    lcp.append(-1) # trailing -1 (at index n)
    return lcp
```

Running time: worst case $O(n^2)$, repetitive texts are bad.
Will be improved in a few minutes by Kasai's algorithm.

Applications of Enhanced Suffix Arrays

Longest Repeated Substring (by Enhanced Suffix Array)

Example

The longest repeated substring in `cabca` is `ca`.

Question

How do we find **longest repeated substring** using the suffix array and LCP array?

Longest Repeated Substring (by Enhanced Suffix Array)

Example

The longest repeated substring in cabca is ca.

Question

How do we find **longest repeated substring** using the suffix array and LCP array?

Answer

- Just look for maximum value in LCP array
- Suffix array at that position tells where the substring starts

Longest Repeated Substring (by Enhanced Suffix Array)

Example

The longest repeated substring in `cabca` is `ca`.

Question

How do we find **longest repeated substring** using the suffix array and LCP array?

Answer

- Just look for maximum value in LCP array
- Suffix array at that position tells where the substring starts
- Running time $O(n)$
- Note that this algorithm is much simple than using the suffix tree.

Example: Longest Repeated Substring via ESA

r	pos[r]	lcp[r]	T[pos[r] :]
0	13	-	$
1	12	0	i$
2	11	1	ii$
3	1	2	iississippii$
4	8	1	ippii$
5	**5**	1	issippii$
6	**2**	**4**	ississippii$
7	0	0	miississippii$
8	10	0	pii$
9	9	1	ppii$
10	7	0	sippii$
11	4	2	sissippii$
12	6	1	ssippii$
13	3	3	ssissippii$

Shortest Unique Substring (Enhanced Suffix Array)

Idea

- For every suffix of $T = s\$$, determine the shortest prefix that is unique; i.e. for each i, determine the smallest j such that $T[i \dots j]$ is a unique substring of T.
- This is easy using the LCP array:

  $$j = i + \max\{\texttt{lcp}[r], \texttt{lcp}[r+1]\}.$$

  where $\texttt{pos}[r] = i$.
- However, we need to exclude cases where $j = n - 1$,
  meaning that $T[i \dots j]$ is only unique due to the sentinel $T[n-1] = \$$.

Code: Shortest Unique Substring

```python
def shortest_unique_substring(pos, lcp):
    n = len(pos)
    # full text (without sentinel) is always unique
    best_i = 0
    best_j = n-1
    for r in range(len(pos)):
        i = pos[r]
        j = i + max(lcp[r], lcp[r+1]) + 1
        if j == n: continue
        if (j-i) < (best_j-best_i):
            best_i, best_j = i, j
    return best_i, best_j
```

Running time: $O(n)$

Longest Common Substrings (using Suffix Arrays)

Problem

Given two strings s, t, find their **longest common substring**.

Example

Let $s = \texttt{ANANAS}$ and $t = \texttt{BANANA}$, then $lcs(s, t) = \texttt{ANANA}$.

Longest Common Substrings (using Suffix Arrays)

Problem

Given two strings s, t, find their **longest common substring**.

Example

Let $s = \texttt{ANANAS}$ and $t = \texttt{BANANA}$, then $lcs(s, t) = \texttt{ANANA}$.

Idea

- Build **generalized** enhanced suffix array of s and t,
  i.e. build the enhanced suffix array $T = s\$_1 t\$_2$.
- Common substring $\rightarrow$ **consecutive positions** in suffix array
- Length given by LCP value
- Distinguish: repeat in one string vs. common substring

Code: Longest Common Substring

```python
def longest_common_substring(s,t):
    T = s + '#' + t + '$'
    pos, lcp = sa_and_lcp(T)
    lcs = ''
    for r in range(1, len(pos)):
        # do both suffixes start in the same string => skip r
        if (pos[r] <= len(s) and pos[r-1] <= len(s)) \
        or (pos[r] > len(s) and pos[r-1] > len(s)):
            continue
        if lcp[r] > len(lcs):
            lcs = T[pos[r]:pos[r]+lcp[r]]
    return lcs
```

Code: Longest Common Substring

```
1 def longest_common_substring(s,t):
2     T = s + '#' + t + '$'
3     pos, lcp = sa_and_lcp(T)
4     lcs = ''
5     for r in range(1, len(pos)):
6         # do both suffixes start in the same string => skip r
7         if (pos[r] <= len(s) and pos[r-1] <= len(s)) \
8         or (pos[r] > len(s) and pos[r-1] > len(s)):
9             continue
10        if lcp[r] > len(lcs):
11            lcs = T[pos[r]:pos[r]+lcp[r]]
12    return lcs
```

Running time: $O(n)$, assuming setting `lcs` in line 11 is $O(1)$

Maximal Unique Matches (MUMs)

Definitions

- Let two strings $s, t \in \Sigma^*$ be given.
- A string u is a **unique match** if it occurs **exactly** once in s and t, respectively.
- A unique match u is **maximal** if there is no $a \in \Sigma$ such that au or ua is a unique match.

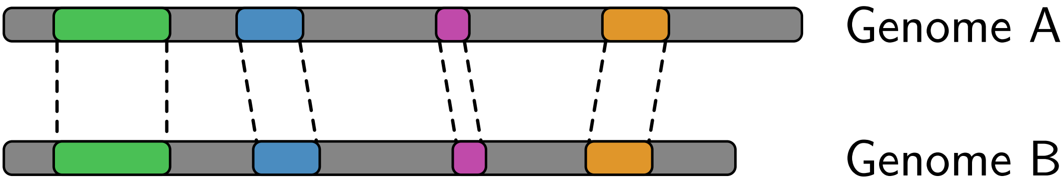
Maximal Unique Matches (MUMs)

Definitions

- Let two strings $s, t \in \Sigma^*$ be given.
- A string u is a **unique match** if it occurs **exactly** once in s and t, respectively.
- A unique match u is **maximal** if there is no $a \in \Sigma$ such that au or ua is a unique match.

Significance of MUMs

MUMs can be used as anchor points for aligning long sequences.



Idea: Computing MUMs using Enhanced Suffix Arrays

Reuse from longest common substrings:

- Build **generalized** enhanced suffix array of s and t, i.e. build the enhanced suffix array $T = s\$_1 t\$_2$.
- Common substring $\rightarrow$ **consecutive positions** in suffix array
- Length given by LCP value
- Distinguish: repeat in one string vs. common substring

Additional considerations

- Ensure hits are unique: "isolated" local maxima in LCP table
- Check that we cannot extend to the left

Example: Computing MUMs

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
A	C	B	B	A	B	A	C	C	C	A	$\$_1$	B	A	B	B	A	B	C	C	A	$\$_2$

r	pos[r]	lcp[r]	r	pos[r]	lcp[r]
0	11	-1	12	5	2
1	21	0	13	2	1
2	10	0	14	14	4
3	20	1	15	17	1
4	4	1	16	9	0
5	13	2	17	19	2
6	16	2	18	1	1
7	0	1	19	8	1
8	6	2	20	18	3
9	3	0	21	7	2
10	12	3	22	-	-1
11	15	3			

Example: Computing MUMs

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
A	C	B	B	A	B	A	C	C	C	A	$\$_1$	B	A	B	B	A	B	C	C	A	$\$_2$

r	pos[r]	lcp[r]	r	pos[r]	lcp[r]
0	11	-1	12	5	2
1	21	0	13	2	1
2	10	0	14	14	4
3	20	1	15	17	1
4	4	1	16	9	0
5	13	2	17	19	2
6	16	2	18	1	1
7	0	1	19	8	1
8	6	2	20	18	3
9	3	0	21	7	2
10	12	3	22	-	-1
11	15	3			

Local maxima

Example: Computing MUMs

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
A	C	B	B	A	B	A	C	C	C	A	$\$_1$	B	A	B	B	A	B	C	C	A	$\$_2$

r	pos[r]	lcp[r]	r	pos[r]	lcp[r]
0	11	-1	12	5	2
1	21	0	13	2	1
2	10	0	14	14	4
3	20	1	15	17	1
4	4	1	16	9	0
5	13	2	17	19	2
6	16	2	18	1	1
7	0	1	19	8	1
8	6	2	20	18	3
9	3	0	21	7	2
10	12	3	22	-	-1
11	15	3			

Local maxima

Example: Computing MUMs

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
A	C	B	B	A	B	A	C	C	C	A	$\$_1$	B	A	B	B	A	B	C	C	A	$\$_2$

r	pos[r]	lcp[r]	r	pos[r]	lcp[r]
0	11	-1	12	5	2
1	21	0	13	2	1
2	10	0	[illegible]	[illegible]	4
3	20	1	[illegible]	[illegible]	1
4	4	1	16	9	0
5	13	2	17	19	2
6	16	2	18	1	1
7	0	1	[illegible]	8	1
8	6	2	[illegible]	3	3
9	3	0	21	7	2
10	12	3	22	-	-1
11	15	3			

Not maximal!

Not unique!

Local maxima

Example: Computing MUMs

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
A	C	B	B	A	B	A	C	C	C	A	$\$_1$	B	A	B	B	A	B	C	C	A	$\$_2$

r	pos[r]	lcp[r]	r	pos[r]	lcp[r]
0	11	-1	12	5	2
1	21	0	13	2	1
2	10	0	14	14	4
3	20	1	15	17	1
4	4	1	16	9	0
5	13	2	17	19	2
6	16	2	18	1	1
7	0	1	19	8	1
8	6	2	20	18	3
9	3	0	21	7	2
10	12	3	22	–	-1
11	15	3			

Not maximal!

Same string!

Local maxima

Example: Computing MUMs

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
A	C	B	B	A	B	A	C	C	C	A	$\$_1$	B	A	B	B	A	B	C	C	A	$\$_2$

r	pos[r]	lcp[r]	r	pos[r]	lcp[r]
0	11	-1	12	5	2
1	21	0	13	2	1
2	10	0	14	14	4
3	20	1	15	17	1
4	4	1	16	9	0
5	13	2	17	19	2
6	16	2	18	1	1
7	0	1	19	8	1
8	6	2	20	18	3
9	3	0	21	7	2
10	12	3	22	-	-1
11	15	3			

Local maxima

Valid MUMs:

CCA BBAB

Code: Computing MUMs

```
def compute_mums(s,t):
    T = s + '#' + t + '$'
    pos, lcp = sa_and_lcp(T)
    for r in range(1, len(pos)):
        p1, p2 = pos[r-1], pos[r]
        if (p1 <= len(s)) and (p2 <= len(s)):
            continue
        if (p1 > len(s)) and (p2 > len(s)):
            continue
        if (lcp[r-1] >= lcp[r]) or
           (lcp[r+1] >= lcp[r]):
            continue
        if (p1 == 0) or (p2 == 0) or
           (T[p1-1] != T[p2-1]):
            yield T[p1:p1+lcp[r]]
```

Constructing LCP Arrays in Linear Time

Inverting the Suffix Array

Observations

- Any suffix array is a **permutation** of numbers from 0 to $n-1$.
- A suffix array can thus be **inverted** (in linear time):

Inverting the Suffix Array

Observations

- Any suffix array is a **permutation** of numbers from 0 to $n-1$.
- A suffix array can thus be **inverted** (in linear time):

Terminology

- **Suffix array:** `pos[r]` is the start **pos**ition of the suffix with lexicographical rank `r`.
- **Inverted suffix array:** `rank[p]` is the lexicographical **rank** of the suffix that starts at position `p`.

Inverting the Suffix Array

Observations

- Any suffix array is a **permutation** of numbers from 0 to $n-1$.
- A suffix array can thus be **inverted** (in linear time):

Terminology

- **Suffix array:** `pos[r]` is the start **pos**ition of the suffix with lexicographical rank `r`.
- **Inverted suffix array:** `rank[p]` is the lexicographical **rank** of the suffix that starts at position `p`.

Linear-time inversion

```
for r in range(n):  rank[pos[r]] = r
```

Note: `rank` is filled in random-access order.

Example: Inverting the Suffix Array

Linear Time LCP Construction: Kasai's Algorithm

Input

Text `T`, suffix array `pos`, its inverse `rank`.

Idea

- Compare each suffix, starting at text position $p = 0, 1, \ldots, n-1$, to its respective predecessor ($=$ lexicographically next smaller suffix)
- Get predecessor by using suffix array (`pos`) and its inverse (`rank`): For the suffix starting at p, find text position $\mathtt{pos}[\mathtt{rank}[p] - 1]$.
- Fill in LCP table in $\mathtt{rank}[p]$-order (not from left to right or r-order!)

Linear Time LCP Construction: Kasai's Algorithm

Input

Text T, suffix array pos, its inverse rank.

Idea

- Compare each suffix, starting at text position $p = 0, 1, \ldots, n-1$, to its respective predecessor (= lexicographically next smaller suffix)
- Get predecessor by using suffix array (pos) and its inverse (rank): For the suffix starting at p, find text position $\texttt{pos}[\texttt{rank}[p] - 1]$.
- Fill in LCP table in $\texttt{rank}[p]$-order (not from left to right or r-order!)
- Moving from p to $p+1$, we keep the computed common prefix, without the first character, similarly to following a suffix link. This is what saves us time.

Example: Kasai's Algorithm

r	pos[r]	lcp[r]	T[pos[r] :]
0	13	-	$
1	12		i$
2	11		ii$
3	1		iissisissippii$
4	8		ippii$
5	5		issippii$
6	**2**		issisissippii$
7	**0**	0	miissisissippii$
8	10		pii$
9	9		ppii$
10	7		sippii$
11	4		sissippii$
12	6		ssippii$
13	3		ssissippii$

Example: Kasai's Algorithm

r	pos[r]	lcp[r]	T[pos[r] :]
0	13	-	$
1	12		i$
2	**11**		ii$
3	**1**	2	iissisissippii$
4	8		ippii$
5	5		issippii$
6	2		ississippii$
7	0	0	miississippii$
8	10		pii$
9	9		ppii$
10	7		sippii$
11	4		sissippii$
12	6		ssippii$
13	3		ssissippii$

Example: Kasai's Algorithm

r	pos[r]	lcp[r]	T[pos[r] :]
0	13	-	$
1	12		i$
2	11		ii$
3	1	2	iississippii$
4	8		ippii$
5	**5**		issippii$
6	**2**	4	ississippii$
7	0	0	miississippii$
8	10		pii$
9	9		ppii$
10	7		sippii$
11	4		sissippii$
12	6		ssippii$
13	3		ssissippii$

Code: Kasai's Algorithm

```python
def lcp(pos, T):
    n = len(pos)
    lcp = [-1] * (n+1)
    rank = invert_sa(pos)
    l = 0 # current common prefix length
    for p in range(n-1):
        r = rank[p]
        # within length of T and chars agree?
        while (pos[r-1] + l < len(T)) and
              (p + l < len(T)) and
              (T[p+l] == T[pos[r-1] + l]):
            l += 1
        lcp[r] = l
        l = max(l-1, 0)  # next suffix: lose first character
    return lcp
```

Why Is That Linear Time?

```
1   for p in range(n-1):
2       r = rank[p]
3       while (pos[r-1] + l < len(T)) and
4             (p + l < len(T)) and
5             (T[p+l] == T[pos[r-1] + l]):
6           l += 1
7       lcp[r] = l
8       l = max(l-1, 0)   # next suffix: lose first character
```

Test in Line 5 can be performed at most $2n$ times:

- Mismatch: `while` loop terminated: at most $n-1$ times
- Match: `l` is incremented in Line 6 and can decrease by at most 1 in Line 8
- p increased in Line 1
  $\rightarrow$ `p+l` is larger when next reaching Line 5
  $\rightarrow$ can happen at most n times

Summary

Today

- **Suffix arrays**
- **LCP array**
- **Enhanced suffix array** can often replace suffix tree
- **Applications**
  - Longest repeated substring
  - Shortest unique substring
  - Longest common substring
  - Maximal unique matches (MUMs)
- Kasai's algorithm: linear time **LCP array** construction

Exam Questions

- Define a suffix array.
- Construct a suffix array for an example string.
- Explain pattern search in suffix arrays.
- Give the definition of the LCP array and explain it.
- Construct the LCP array for a given string.
- What is the advantage of an enhanced suffix array over a suffix tree?
- Explain the following problems and how they can be solved using an enhanced suffix array: longest repeated substring, shortest unique substring, longest common substring, maximal unique matches.
- Why and how can a suffix array be inverted?
- Explain Kasai's algorithm. What is its running time?
- Apply Kasai's algorithm to a given example.